

5.2 One-to-One Functions; Inverse Functions

One-to-one Function: A function is *one-to-one* if for any value of x there is exactly one y (otherwise it wouldn't be a function), and for any value of y , there is exactly one x .

Example: Determine whether the following functions are one to one.

a) $\{(-2, 6), (-1, 3), (0, 2), (1, 5), (2, 8)\}$

b) $\{(1, 1), (2, 4), (3, 9), (0, 0), (-1, 1), (-2, 4)\}$

Horizontal Line Test: If every horizontal line intersects the graph of a function f in at most one point, then f is one-to-one.

Example: For each function, use its graph to determine whether the function is one-to-one.

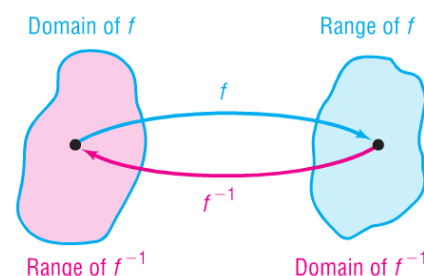
a) $f(x) = |x|$

b) $g(x) = \sqrt[3]{x}$

Theorem: A function that is increasing on an interval I is a one-to-one function on I .

A function that is decreasing on an interval I is a one-to-one function on I .

Inverse Function: Two functions are *inverses* if and only if whenever one function contains the element (a, b) , the other function contains the element (b, a) . If f is a one-to-one function, the correspondence from the range of f back to the domain of f is called the *inverse function* of f . The inverse of f is abbreviated f^{-1} .



★ **Domain of f = Range of f^{-1}**

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Example: Find the inverse of the following one-to-one function: $\{(2, 3), (4, 5), (6, 8), (9, 10), (12, 14)\}$

If we start with x , apply f , and then apply f^{-1} , we get x back again.

If we start with x , apply f^{-1} , and then apply f , we get x back again.

What f does, f^{-1} undoes, and vice versa. In other words,

$$f^{-1}(f(x)) = x, \text{ where } x \text{ is in the domain of } f.$$

$$f(f^{-1}(x)) = x \text{ where } x \text{ is in the domain of } f^{-1}.$$

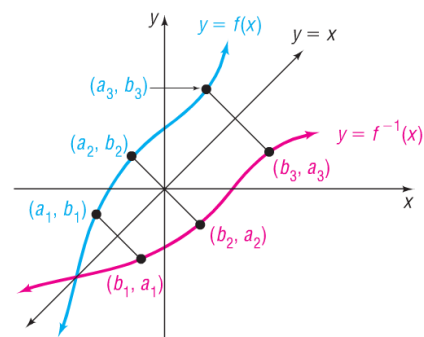
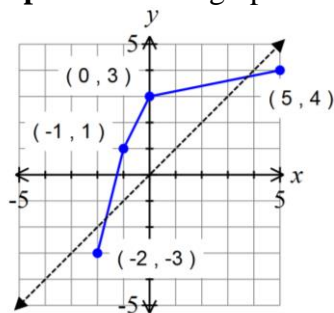
To verify that two functions are inverses, show that $f^{-1}(f(x)) = x$ and $f(f^{-1}(x)) = x$

Example: Verify that the inverse of $f(x) = \frac{2}{x+5}$ is $f^{-1}(x) = \frac{2}{x} - 5$.

Example: Verify that the inverse of $f(x) = \sqrt[3]{2x}$ is $f^{-1}(x) = \frac{x^3}{2}$.

Theorem: The graph of a function f and the graph of its inverse f^{-1} are symmetric with respect to the line $y = x$.

Example: Draw the graph of the inverse function.



Finding the Inverse of a Function

1. Rewrite $f(x)$ as y in the original equation.
2. Interchange x and y .
3. Solve for y .
4. Replace y with the notation $f^{-1}(x)$.

Example: Find the inverse. State the domain and range of $f(x)$ and the domain and range of $f^{-1}(x)$.

a) $f(x) = -3x + 1$

b) $f(x) = \frac{2x+3}{5x-4}$